

Maximum lepton asymmetry from active – sterile neutrino oscillations in the Early Universe.

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Abstract

A large lepton asymmetry could be generated in the Early Universe by oscillations of active to sterile neutrinos with a small mixing angle $\sin 2\theta < 10^{-2}$. The final order of magnitude of the lepton asymmetry η is mainly determined by its growth in the last stage of evolution when the MSW resonance dominates the kinetic equations. In this paper we present a simple way of calculating the maximum possible lepton asymmetry which can be created. Our results are in good agreement to previous calculations. Furthermore, we find that the growth of asymmetry does not obey any particular power law. We find that the maximum possible asymmetry at the freeze-out of the n/p ratio at $T \sim 1$ MeV strongly depends on the mass-squared difference δm^2 : the asymmetry is negligible for $|\delta m^2| \ll 1 \text{ eV}^2$ and reaches asymptotically large values for $|\delta m^2| \gtrsim 50 \text{ eV}^2$.

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1 Introduction

Neutrino oscillations between muon and tau neutrinos, which were found recently in the Super-Kamiokande experiment [1], prove the existence of neutrino masses. This finding does not spoil the Standard Big Bang Nucleosynthesis (BBN) picture because experimentally interesting neutrino masses for all three flavors ν_i , $i = e, \mu, \tau$, are small $m_i \leq 1$ eV, in comparison to typical BBN temperatures $T \sim 1$ MeV. As for oscillations between active flavors, these do not play any role in BBN because all active flavors are equally populated.

However, if sterile neutrinos exist, oscillations between active and sterile states could have significant consequences for BBN. One possibility is that the total energy density of the Universe is changed due to excited sterile neutrinos; together with a recent analysis of observational data [2], which claims that the effective number of neutrinos can not exceed 3.2, this effect can be used to exclude some region of the $(\delta m^2, \sin 2\theta)$ parameter plane e.g. for heavy sterile neutrinos [3].

Another possible influence of sterile neutrinos on BBN arises from the fact that ν_e - ν_s oscillations can create a large asymmetry between ν_e and $\bar{\nu}_e$ which would have a direct impact on the n - p reactions and thus change the light element abundances. This case is interesting for small negative δm^2 and small $\sin 2\theta < 10^{-2}$, and the important result will be the value of the electron neutrino asymmetry at the freeze-out of the n/p ratio at a temperature around 1 MeV. Actually, the magnitude of ν_e asymmetry at $T = 1$ MeV is just an indicator for possible consequences on BBN. For accurate calculations one would need to use exact non-equilibrium distribution functions of ν_e and $\bar{\nu}_e$ in the neutron-proton reactions. A general discussion of this subject can be found in Section 2.1 of [4], and details in case of large asymmetry in [5].

In early works with simplified calculations it was found that the asymmetry could not reach large final values [6, 7]. However, later it was found that the asymmetry can increase significantly and reach values $\mathcal{O}(0.1)$ [8]. This finding was confirmed by elaborate numerical calculations [9, 10, 11]. A main feature of these results was

that the asymmetry grows according to the power law T^{-4} . A contradictory result that the power law was T^{-1} was suggested in [12], but was found to be erroneous in [13, 14].

In the present paper we investigate the evolution of the lepton asymmetry under the assumption that all active neutrinos passing the resonance are converted to sterile neutrinos, thus forming a Fermi-Dirac distribution in the sterile sector. This assumption is correct in the case of pure Mikheyev-Smirnov-Wolfenstein (MSW) transition [15], i.e. when one can neglect the collision-integral terms. Although the assumption of full conversion from active to sterile states may be too strong for certain models, our result does present the maximum possible asymmetry. The advantage of our approach is that we do not solve any differential equations. Thus, our approach is extremely simple and safe. Furthermore, we consider the effects of various simplifications and compare our results with previous works, in particular with [9]. Finally, we investigate how the maximum amplitude of the asymmetry at the temperature 1 MeV depends on initial conditions and δm^2 , and compare it with a more detailed calculation, [16].

2 Maximal asymmetry at given temperature

We are considering neutrino oscillations in the Early Universe at temperatures around $1 \text{ MeV} \leq T \leq 100 \text{ MeV}$. In this epoch, the Universe is a homogeneous and isotropic plasma, and the expansion is described by the Hubble parameter,

$$H = \sqrt{\frac{8\pi\rho_{\text{tot}}}{3m_{\text{pl}}^2}}, \quad (1)$$

where $m_{\text{pl}} = 1.22 \times 10^{22} \text{ MeV}$ is the Planck mass. The total energy density is

$$\rho_{\text{tot}} = \frac{\pi^2}{30} g_* T^4, \quad (2)$$

where $g_* = 10.75$ is the number of effective degrees of freedom during the epoch of interest. We also suppose that $\dot{T}/T = -H$, which is a good approximation for the models we are considering here.

The neutrino oscillations are described by the density matrix formalism, see for instance [17]. However, we will mainly be interested in the resonance condition, which for $\nu_a \leftrightarrow \nu_s$ oscillations is given by [18]

$$\frac{\delta m^2 \cos 2\theta}{2E} = -V_{\text{eff}}^a = -\left(\mp C_1 \eta G_F T^3 + C_2^a \frac{G_F^2 T^4 E}{\alpha}\right), \quad (3)$$

where $a = e, \mu, \tau$ specifies the type of active neutrino, $\delta m^2 = m_s^2 - m_a^2$ is the mass squared difference, θ is the mixing angle, and V_{eff}^a is the effective potential. The “ \mp ” signs refer to neutrinos and anti-neutrinos, respectively. Furthermore, $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant, $\alpha = 1/137$ is the fine structure constant, $C_1 = 0.345$, $C_2^e = 0.61$ for ν_e - ν_s mixing, $C_2^{\mu, \tau} = 0.17$ for $\nu_{\mu, \tau}$ - ν_s and T is the temperature of the plasma. Sometimes we will use the notation $\delta m_{eV}^2 = \delta m^2/\text{eV}^2$. Finally, the effective asymmetry is $\eta = 2\eta_{\nu_a} + \eta_0$, where η_{ν_a} is the neutrino asymmetry of the active species a , and η_0 is some small fermion asymmetry from all other fermion species. For our analysis it is only interesting that η_0 is of order of the baryon asymmetry, 10^{-10} .

Because the temperature is much larger than the neutrino masses, not only neutrino momentum and energy are equal, $p = E$, but also that a certain neutrino mode evolves with constant $y \equiv p/T$. We will therefore use this dimensionless variable instead of momentum and energy, so $p = E = yT$.

We briefly discuss the evolution of the neutrino densities for small mixing angles, $\sin 2\theta < 10^{-2}$, looking at ν_e for definiteness. Let us first sketch the reason why the lepton asymmetry can change at all. Due to the mixing of ν_e with ν_s , the number densities of the ν_e and $\bar{\nu}_e$ can alter. The sterile neutrinos that are thus created have no effect on η . Now if $\eta \neq 0$ initially, the resonance condition for the ν_e and $\bar{\nu}_e$ will be different. Therefore their number densities will change differently, thereby changing η . Thus neutrino mixing works as a back-reaction on η . The actual change in asymmetry and density takes place close to the resonance momentum, which is defined by Eq. (3),

$$y_{\text{res}} = \sqrt{\left(k_1 \frac{\eta}{T^2}\right)^2 + k_2 \frac{1}{T^6}} \pm k_1 \frac{\eta}{T^2}, \quad (4)$$

where

$$k_1 = \frac{C_1}{2C_2^e G_F / \alpha} \quad \text{and} \quad k_2 = \frac{|\delta m^2| \cos 2\theta}{2C_2^e G_F^2 / \alpha}. \quad (5)$$

There are three different stages of lepton asymmetry evolution. During the first stage the asymmetry is so small that the k_1 terms can be neglected in Eq. (4). The neutrinos and the anti-neutrinos simultaneously pass through the resonance. Furthermore, any initial asymmetry decreases due to the negative back-reaction.

The second stage is the stage of exponential growth of the lepton asymmetry, which occurs when the back-reaction becomes positive. Here, the resonance conditions for ν and $\bar{\nu}$ are rapidly driven apart. For definiteness, we will assume that the created asymmetry is positive, $\eta > 0$. Then y_{res} for the ν increases to very high momenta where the Fermi distribution is negligible, the resonance momentum for $\bar{\nu}$ decreases to small momenta $y_{\text{res}} < 1$. At the same time, the asymmetry grows many orders of magnitude and reaches values of order of 10^{-6} .

In the third stage the lepton asymmetry grows at a slower rate. There has been a lot of controversy on this subject, and it has often been proposed that the lepton asymmetry would grow according to an approximate power law, $\eta \propto T^{-\alpha}$, where $\alpha = 1-4$.

We will now show that any power law is only an approximation to the true asymmetry growth. Our considerations start at the end of the exponential regime, which happens at temperature T_i . The important variables are the asymmetry $\eta_i(T_i)$ and the resonance momentum $y_i(T_i, \eta_i)$ at this time. For definiteness, we will assume $\eta_i > 0$, so that we can safely neglect ν oscillations (with $\eta_i < 0$ we could neglect $\bar{\nu}$).

The resonance momentum is governed by two effects: it decreases for increasing η and increases for decreasing T . At the end of the exponential regime, these two effects compensate each other, so that at first the resonance momentum changes very slowly. This plus the fact that collisional damping is small may let us assume that the momentum modes passing the resonance in the regime of slow asymmetry growth will fully excite the sterile sector. For negligible collisional damping, this is exactly

the MSW effect [15]. The full excitation of the sterile sector at resonance has also been stated in [9].

We can now calculate the asymmetry for a given temperature $T < T_i$:

$$\eta(T) = \eta_i + \frac{1}{2\zeta(3)} \int_{y_i}^{y_{\text{res}}(T)} dy y^2 f_{\text{eq}}(y, \tilde{\mu}) . \quad (6)$$

Here, f_{eq} is the equilibrium Fermi distribution function, given by

$$f_{\text{eq}}(y, \tilde{\mu}) = \frac{1}{e^{y-\tilde{\mu}} + 1} , \quad (7)$$

where $\tilde{\mu} = \mu/T$ is the dimensionless chemical potential for the active neutrinos. For $\eta \ll 1$ and large temperatures $T \gg 1$ MeV, the chemical potential is approximately $\tilde{\mu} \approx -1.5\eta$. Therefore, we can safely neglect it. We will discuss its influence for large η later on. We should mention that η_i has its origin in an already weakly excited sterile sector at momenta $0 < y < 2$. However, the correction to Eq. (6) from this excitation is negligible.

We can now insert Eq. (4) into Eq. (6) and solve the equation for $\eta(T)$. The resulting equation only depends on the input parameters η_i and y_i , and on the neutrino parameter δm^2 . Interestingly, the equation is independent of the mixing angle $\sin 2\theta$, a fact which stems from the full transition assumption. The result has another striking feature: even if the sterile sector is not fully excited during resonance, which happens if the resonance momentum changes too fast and/or the collisional damping is important, our result still gives the maximally possible asymmetry at temperature T .

For small momenta, the Fermi-Dirac distribution can be approximated by $f_{\text{eq}} = 1/2$. Then Eq. (6) simplifies to

$$\eta(T) - \eta_i = \frac{1}{12\zeta(3)} \left(y_{\text{res}}^3(T) - y_i^3 \right) . \quad (8)$$

Furthermore, for large asymmetries and small temperatures, the k_2 term in Eq. (4) is much smaller than the k_1 term, so that

$$y_{\text{res}} \simeq \frac{k_2}{2k_1\eta T^4} . \quad (9)$$

Then

$$\eta^4 + \eta^3 \left(\frac{1}{12\zeta(3)} y_i^3 - \eta_i \right) = \frac{1}{12\zeta(3)} \left(\frac{k_2}{2k_1 T^4} \right)^3. \quad (10)$$

This equation can be easily solved in two limits: when $\eta \ll (\frac{1}{12\zeta(3)} y_i^3 - \eta_i)$ we can neglect the first term on the LHS so that we obtain a $\eta \propto T^{-4}$ power law. However, when $\eta \gg (\frac{1}{12\zeta(3)} y_i^3 - \eta_i)$ the solution is $\eta \propto T^{-3}$. Thus the power law will slowly change in time.

This is also what we expect from the following considerations: the asymmetry is created close to the resonance momentum. However, when the sterile sector is fully excited at this momentum, the asymmetry can no longer be created. Naturally, when η does not change, the resonance momentum will increase due to the decreasing temperature. Thus we expect a slowly increasing resonance momentum. From Eq. (9) we can then deduce that η increases more slowly than a $\eta \propto T^{-4}$ power law.

We can easily derive an equation similar to Eq. (14) of [9] by differentiating Eq. (6) in the temperature T :

$$\frac{d\eta}{dT} = \frac{1}{2\zeta(3)} y_{\text{res}}^2(T) f_{\text{eq}}(y_{\text{res}}, 0) \frac{dy_{\text{res}}}{dT}. \quad (11)$$

The only difference between our Eq. (11) and Eq. (14) of [9] is in the usually negligible term proportional to f_{ν_s} . Furthermore, Eq. (18) of [9], which can be derived from their Eq. (14) directly, is in our notation

$$\frac{d\eta}{dT} = \frac{-4 \frac{y^3}{2\zeta(3)} f_{\text{eq}}}{T + \frac{y^3}{2\zeta(3)} f_{\text{eq}} T / \eta}, \quad (12)$$

where we have used the simplification Eq. (9).

For illustration, we have plotted the evolution of the “power” α which can be derived by the formula

$$\alpha = \frac{T}{\eta} \frac{d\eta}{dT} \quad (13)$$

in Fig. 1. Here, we have already included the effect from the exact Fermi-Dirac distribution, which becomes important at low temperatures. We have plotted the evolution for $\delta m^2 = 1 \text{ eV}^2$ and for different values of y_i . Note that typical values are

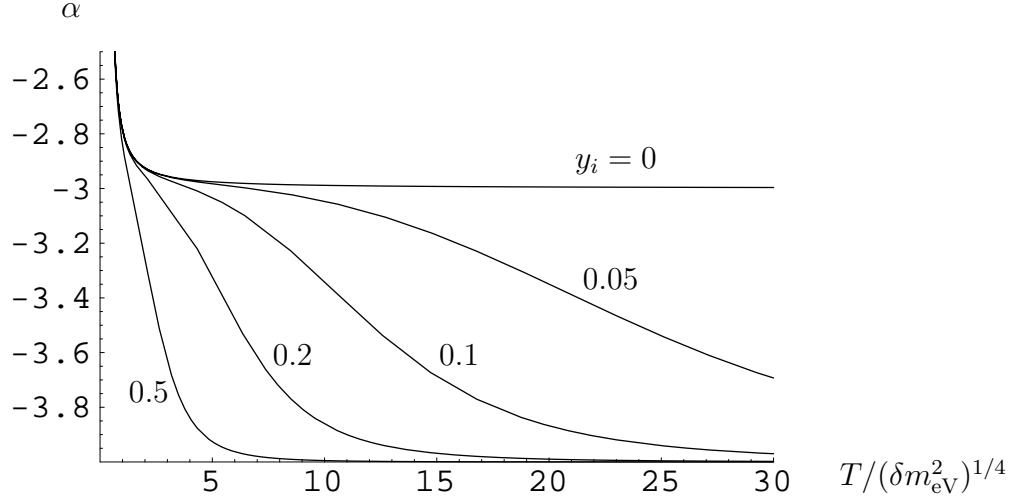


Figure 1: Power-law index α as a function of temperature for various values of the initial resonance momentum y_i . We have used the simplified condition for the resonance momentum and have neglected the initial asymmetry η_i . Note that e.g. $\eta_i \sim 6 \times 10^{-6}$ would substantially change the line for $y_i \leq 0.1$.

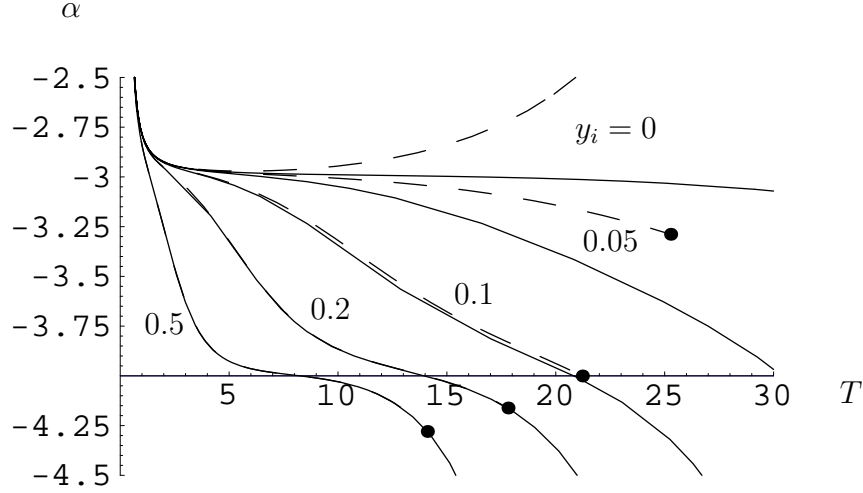


Figure 2: Power-law index α as a function of temperature for various values of the initial resonance momentum y_i . Here we have used the exact resonance momentum condition. The solid lines represent the case $\eta_i = 0$, whereas the dashed lines show the effect from non-zero η_i , in this case $\eta_i = 6 \times 10^{-6}$. The dots show starting-points, i.e. T_i and the corresponding power α_i (for $\eta_i = 0$, $T_i = \infty$).

$y_i = 0.1-0.2$. Furthermore, we have set $\eta_i = 0$. Typical values would be $\eta_i = 10^{-6}-10^{-5}$, but as we can see from Eq. (10), such values are negligible in comparison with the y_i^3 term. Anyhow, we can simulate the effect of η_i by changing y_i , see Eq. (10). Interestingly, the effect from δm^2 can be compensated exactly by rescaling the temperature:

$$T_{\delta m^2} = (\delta m^2)^{1/4} T_{\delta m^2=1 \text{ eV}^2}. \quad (14)$$

To be still more precise, we need to use the exact Eq. (4) instead of Eq. (9). We have plotted the results in Fig. 2 in analogy to Fig. 1. We see that both the approximate and the exact resonance condition give the same results at low temperatures, while at high temperatures the results are very different; the exact result shows a larger power-law behavior. This means that there is no real power-law solution in the MSW dominated region, at least under the assumption that transition is complete. We should mention that for $\nu_{\mu,\tau}-\nu_s$ oscillations, the correction due to the exact resonance condition is somewhat weaker, since C_2 is smaller by a factor 4.

We should note that using Eq. (9) instead of Eq. (4) has no influence on the later evolution of the asymmetry, i.e. Eq. (9) is a good approximation. This is because the apparently larger increase of η for given T_i , η_i is compensated by a smaller y_i when using Eq. (4).

In Fig. 2 we also show the effect of a non-zero initial asymmetry. This effect is important only for small initial values of the resonance momentum $y_{\text{res}} < 0.1$.

3 Asymmetry at the freeze-out of the n/p ratio.

If we want to investigate the influence of the neutrino asymmetry on BBN, we need to check its value at the temperature $T = 1 \text{ MeV}$ when the neutron-proton reactions decouple and the n/p ratio freezes out. In Fig. 3 we present the dependence of the maximum possible asymmetry at the temperature 1 MeV from $|\delta m_{\text{eV}}^2|$. The solid line shows the maximum asymmetry for the initial conditions $y_i = 0$ and $\eta_i = 0$. Our result is in good agreement with previous extensive calculations [16]. Naturally, we

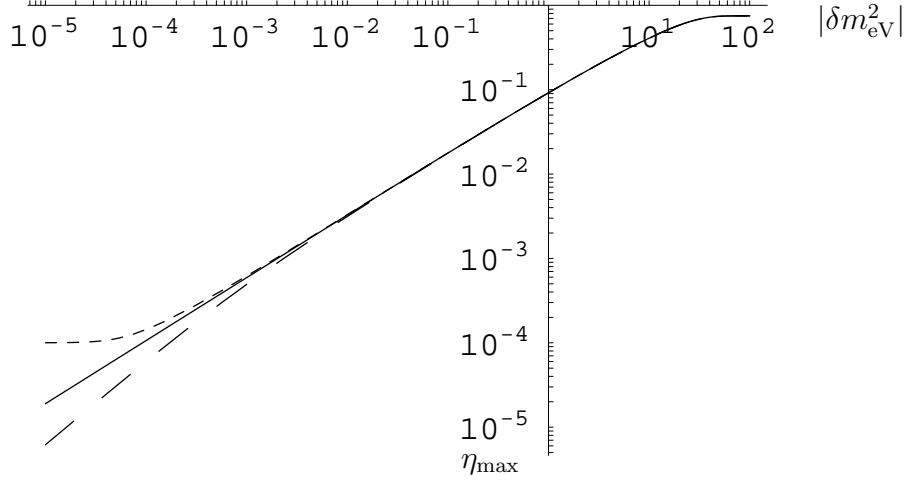


Figure 3: The maximal possible asymmetry η at the freeze-out temperature of the n - p interactions, $T = 1$ MeV, as a function of $|\delta m_{eV}^2|$, for $\eta_i = y_i = 0$ (solid line). The initial asymmetry η_i increases the final asymmetry for small $|\delta m_{eV}^2|$ (short dashed line for $\eta_i = 10^{-4}$); the momentum y_i decreases the asymmetry for small $|\delta m_{eV}^2|$ (long dashed line is for $y_i = 0.2$).

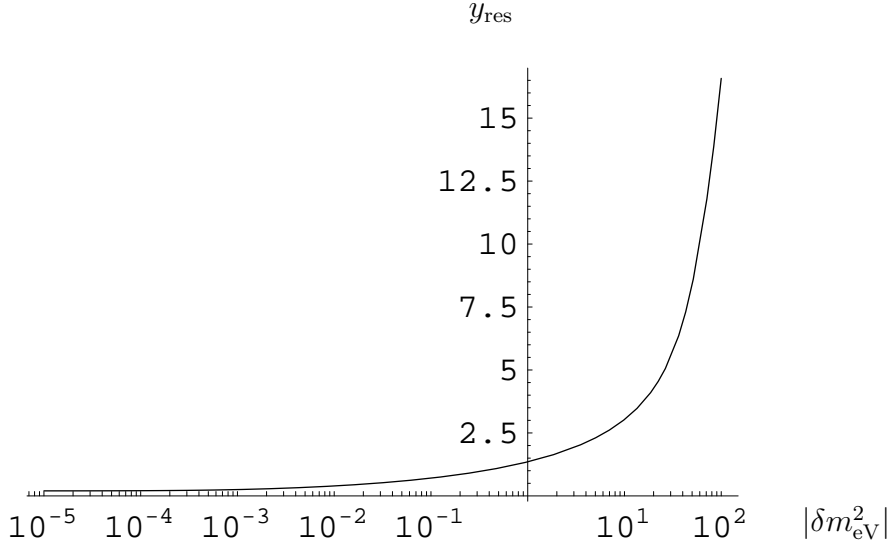


Figure 4: The resonance momentum y_{res} at the freeze-out temperature of the n - p interactions, $T = 1$ MeV, as a function of $|\delta m_{eV}^2|$. The initial asymmetry η_i and momentum y_i only have significant effects for very low $|\delta m_{eV}^2|$; we have neglected them in this plot.

do not expect any dependence on $\sin 2\theta$, since we have assumed full MSW transition. We should therefore stress that our result is only a good approximation in regions of the parameter space $(\delta m^2, \sin 2\theta)$ where there is an effective exponential increase of the asymmetry, and if there is full MSW transition afterwards.

From Fig. 3 we can easily extract that for $|\delta m_{\text{eV}}^2| \ll 1$ the asymmetry will remain small, while for $|\delta m_{\text{eV}}^2| \gg 1$ it can reach its asymptotic value $3/8$, which was found in [9] (the factor 2 difference with our $6/8$ on Fig. 3 stems from the relation $\eta = 2\eta_{\nu_e}$). For $|\delta m_{\text{eV}}^2| < 1$, the asymmetry at temperature 1 MeV can be fitted according to the simple relation

$$\eta = 0.1 \times |\delta m_{\text{eV}}^2|^{3/4} \quad (15)$$

in good agreement with the result of [16], $\eta = 0.1 \times |\delta m_{\text{eV}}^2|^{2/3}$.

We will shortly discuss minor effects on the result. The short dashed line in Fig. 3 shows the effect of a non-zero initial asymmetry (for $\eta_i = 10^{-4}$ in this particular case). A non-zero initial asymmetry will increase the asymmetry at $T = 1$ MeV for small $|\delta m^2|$. A non-zero momentum y_i will work in the opposite direction and decrease the final asymmetry for small $|\delta m_{\text{eV}}^2|$ (long dashed line is for $y_i = 0.2$). Note that for very small $|\delta m_{\text{eV}}^2| \ll 10^{-5}$ the exponential regime is to be expected at $T \sim 1$ MeV so that our results become invalid. Finally, the chemical potential plays an important role when the asymmetry is large. However, its effect is to exponentially damp the number density of the $\bar{\nu}_e$ (in the case $\eta > 0$), thus decreasing the asymmetry. Therefore, our result remains the maximum possible asymmetry. We have estimated the decrease of the asymmetry due to the chemical potential to be less than a factor of 2.

For illustration, Fig. 4 shows the dependence of the dimensionless resonance momentum $y_{\text{res}} = p_{\text{res}}/T$ at the temperature $T = 1$ MeV from the parameter $|\delta m_{\text{eV}}^2|$. We see that at $|\delta m_{\text{eV}}^2| \ll 1$ the resonance momentum is small, while for $|\delta m_{\text{eV}}^2| > 10$ it is already on the tail of the distribution function. This explains the values of the asymmetry presented in Fig. 3.

4 Conclusions

In this work we have found the maximum lepton asymmetry which can be created at a given temperature due to active-sterile neutrino oscillations in the Early Universe. We have assumed full transitions of active to sterile neutrinos due to MSW resonance to derive a very simple relation between the lepton asymmetry and the temperature. From this relation, we found that the evolution of the lepton asymmetry does not evolve according to a specific power law.

We have also investigated on the maximum asymmetry at the temperature 1 MeV when the n/p ratio freezes out. We found that the value of the asymmetry at this time depends on δm^2 , but not on $\sin 2\theta$ in concordance with [16]. Furthermore, the initial settings described by the initial resonance momentum y_i and asymmetry η_i have only an influence for very small δm^2 . This means that details of the previous evolution are not very important for the value of the asymmetry at $T = 1$ MeV. In the parameter region which is related to a large final asymmetry, $\delta m_{\text{eV}}^2 > 10$, our asymmetry values are in good agreement with those of [9].

We stress again that our results depend on the assumption of full transitions of active to sterile neutrinos. If in some part of the parameter space the transition is not total, then the asymmetry can be somewhat smaller. Also, for large asymmetries, the exact value of η may be smaller due to effects from the chemical potential.

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